

NASA Contractor Report 4430

1N-07  
72195  
P.17

# Computer Code for Preliminary Sizing Analysis of Axial-Flow Turbines

Arthur J. Glassman

GRANT NAG3-1165  
FEBRUARY 1992

(NASA-CR-4430) COMPUTER CODE FOR  
PRELIMINARY SIZING ANALYSIS OF AXIAL-FLOW  
TURBINES Final Report (Toledo Univ.) 17 p

CSCD 21E

H1/07

N92-20196

Unclass  
0072195

**NASA**

NASA Contractor Report 4430

# Computer Code for Preliminary Sizing Analysis of Axial-Flow Turbines

Arthur J. Glassman  
*University of Toledo*  
*Toledo, Ohio*

Prepared for  
Lewis Research Center  
under Grant NAG3-1165



National Aeronautics and  
Space Administration

Office of Management

Scientific and Technical  
Information Program

1992

## Summary

This report presents a computer program for the preliminary sizing analysis of axial-flow turbines. The computations are based on mean-diameter flow properties and a stage-average velocity diagram. Given as input to the program are power or pressure ratio, mass flow rate, inlet temperature and pressure, rotative speed, inlet and exit diameters (either hub, mean, or tip), exit radius ratio or stator exit angle, turbine loss coefficient, and gas properties. Computations are then performed for any specified number of stages and for any of three types of velocity diagrams (symmetrical, zero exit swirl, or impulse) or for any specified stage swirl split. Exit turning vanes can be included in the design. The program output includes inlet and exit annulus dimensions, exit temperature and pressure, total and static efficiencies, flow angles, and last-stage absolute and relative Mach numbers.

The analysis and code presented herein are modifications of those in reference 1. New features added to improve modeling rigor and extend code applicability include a generalized velocity diagram, a more flexible meanline path, a reheat model, a radial component of velocity, and a computation of free-vortex hub and tip velocity diagrams. Also, a loss-coefficient calibration was performed to provide recommended values for current technology airbreathing engine turbines.

The analysis method and a complete description of input and output are presented in this report. Sample cases are included to illustrate the use of the program.

## Introduction

The preliminary analysis of a power or propulsion system involves many repetitive calculations to determine system performance, component performance, and component geometries over a range of conditions. This must be done in order to eventually determine the best system and operating conditions. For this type of screening analysis, complete design accuracy and detail for the components are not necessary. Approximate and rapid generalized procedures, rather than complex and time-consuming detailed design procedures, are sufficient to yield the desired component overall geometry and performance characteristics.

A computer program for the preliminary sizing analysis of axial-flow turbines was presented in reference 1. This meanline analysis uses the efficiency computation procedure of refer-

ence 2, which is based on a stage-average velocity diagram, and incorporates assumptions consistent with turbine technology from about 1970, when the program was developed. With turbines for some current and future propulsion systems exhibiting higher pressure ratios and more highly inclined flowpaths than in the past, it became apparent that modifications were required to improve modeling rigor and extend code applicability. New features added to accomplish this include a generalized velocity diagram, a more flexible meanline flowpath, a radial component of velocity, and the computation of free-vortex hub and tip velocity diagrams. Also, a loss-coefficient calibration was performed to provide recommended values for current technology airbreathing engine turbines.

The analysis method and a complete description of input and output are presented in this report. Sample cases are included to illustrate use of the program. Because reference 1 is no longer readily available, this report describes the entire analysis, not only the modifications.

## Symbols

$A$	stage loss parameter
$A_{an}$	annulus area, $m^2$ ; $ft^2$
$C$	blade loss parameter
$C_A$	dimensional constant, $2\pi$ rad/rev; 60 sec/min
$C_B$	dimensional constant, 1; $550 \text{ ft} \cdot \text{lb/sec} \cdot \text{hp}$
$c_x$	axial chord, m; ft
$c_p$	heat capacity, joules/kg $\cdot$ K; Btu/lb $\cdot$ $^{\circ}$ R
$D$	diameter, m; ft
$E$	squared ratio of stage-exit axial velocity to stage-average axial velocity
$F$	blade loss weighting factor
$g$	dimensional constant, 1; $32.2 \text{ ft} \cdot \text{lbm/sec}^2 \cdot \text{lb}$
$\Delta h$	specific work, J/kg; Btu/lb
$i$	stage number, $i = 1, 2, \dots, n$
$J$	dimensional constant, 1; $778 \text{ ft} \cdot \text{lb/Btu}$
$j$	stage number for change in meanline slope
$K$	turbine loss coefficient
$k$	coefficients for axial chord correlation
$M$	Mach number

$N$	rotative speed, rad/sec; rpm
$n$	number of stages
$P$	shaft power, W; hp
$p$	pressure, N/m <sup>2</sup> ; lb/ft <sup>2</sup>
$R$	gas constant, J/kg K; ft · lbf/lbm °R
$Re$	Reynolds number
$r$	radius, m; ft
$T$	temperature, K; °R
$U$	blade speed, m/sec; ft/sec
$V$	absolute gas velocity, m/sec; ft/sec
$W$	relative gas velocity, m/sec; ft/sec
$w$	mass flow rate, kg/sec; lb/sec
$\alpha$	absolute-flow angle from axial direction, deg
$\beta$	relative-flow angle from axial direction, deg
$\gamma$	specific heat ratio
$\eta$	static efficiency
$\eta'$	total efficiency
$\theta$	meanline angle from horizontal, deg
$\mu$	viscosity, N · sec/m <sup>2</sup> ; lb/sec · ft
$\rho$	density, kg/m <sup>3</sup> ; lb/ft <sup>3</sup>
$\psi$	work factor

#### Subscripts:

$a$	first stage
$abs$	absolute
$cr$	critical
$ev$	exit vane
$ex$	turbine exit
$h$	hub
$i$	stage $i$ , $i = 1, 2, \dots, n$
$id$	ideal
$in$	turbine inlet
$is$	value at turbine inlet entropy
$m$	mean section
$me$	meridional component
$n$	last stage
$rel$	relative
$ro$	rotor
$st$	stator
$t$	tip
$u$	tangential component
$x$	axial component

0	stator inlet
1	stator exit
2	rotor exit

#### Superscripts:

—	turbine overall
'	absolute total condition
"	relative total condition

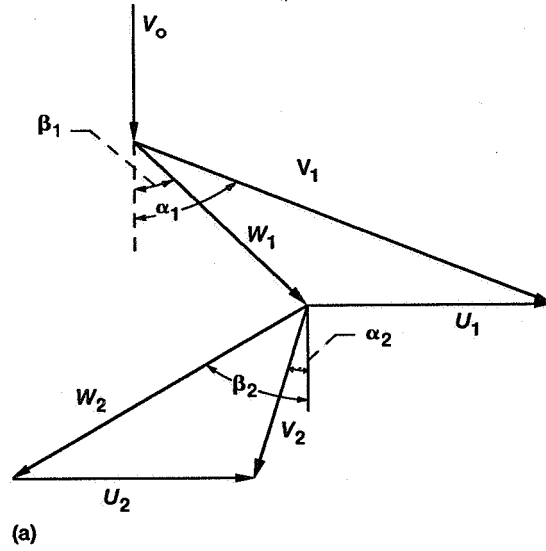
## Method of Analysis

The method is based upon an analysis of the flow at the turbine mean diameter. Radial gradients of the flow properties are not included in the flow and efficiency analyses. However, free-vortex hub and tip velocity-diagram parameters are computed in order to indicate the possible severity of flow conditions at the endwalls. The specific heat ratio is assumed constant throughout the turbine. For any given turbine, all stages, except the first, are specified to have the same shape velocity diagram. The first stage differs only in that the inlet flow can be at any specified flow angle. The velocity diagram shape depends upon the stage work factor value and the specified velocity diagram. Three specific types of velocity diagram (symmetrical, zero exit swirl, and impulse) or a diagram with a specified swirl split (i.e., distribution of swirl velocity between rotor inlet and rotor exit) can be used. Figure 1 illustrates the velocity diagram symbolism and presents the three specific types of velocity diagram for three values of stage work factor. The fundamentals of velocity diagrams and of the flow and loss modeling used herein can be found in reference 3.

Various input options dictate the exact nature of the calculation procedure. There is, however, one basic procedure that is direct and without iteration. This basic procedure will be presented, and then the alternative procedures required for the various input options will be discussed.

### Basic Calculation Procedure

The required inputs for the basic procedure are shaft power  $P$ , mass flow rate  $w$ , inlet total temperature  $T'_{in}$ , inlet total pressure  $p'_{in}$ , rotative speed  $N$ , inlet mean diameter  $D_{m,in}$ , exit mean diameter  $D_{m,ex}$ , stator exit angle  $\alpha_1$ , gas constant  $R$ , specific heat ratio  $\gamma$ , viscosity  $\mu$ , loss coefficient  $K$ , and the squared ratio of stage-exit to stage-average (i.e.,  $(V_{me,0} + V_{me,1} + V_{me,2})/3$ ) meridional velocities  $E$ . Also specified for each calculation are the number of stages  $n$  and the type of velocity diagram. For a multistage turbine, the input variable specified as inlet diameter is used to calculate first rotor blade speed and annulus dimensions at the first rotor exit. Therefore, it is truly an inlet diameter only if the hub and tip diameters



Work factor	Diagram type		
	Zero exit swirl	Impulse	Symmetrical
4.0			
2.0			
1.0			

(b)

(a) Symbolism.

(b) Effect of diagram type and work factor on velocity diagram shape.

Figure 1.—Velocity diagrams (constant axial velocity).

are assumed constant across the first stage. For a one-stage turbine, the specified exit diameter is used for the calculations, and the inlet diameter is of no significance.

The first- and last-stage blade speeds are

$$U_a = \frac{\pi N D_{m,in}}{C_A} \quad (1)$$

$$U_n = \frac{\pi N D_{m,ex}}{C_A} \quad (2)$$

For more than two stages, there are three options for stage blade-speed variation (i.e., mean flowpath):

(1) Linear variation between first and last stages

$$U_i = \frac{U_n - U_a}{n - 1} (i - 1) + U_a \quad (3a)$$

(2) Constant from first stage to  $j^{\text{th}}$  stage ( $i = 1$  to  $j$ )



$$U_i = U_a \quad (3b-1)$$

and then linear to last stage ( $i = j$  to  $n$ )

$$U_i = \frac{U_n - U_a}{n - j} (i - j) + U_a \quad (3b-2)$$

(3) Linear from first stage to  $j^{\text{th}}$  stage ( $i = 1$  to  $j$ )

$$U_i = \frac{U_n - U_a}{j - 1} (i - 1) + U_a \quad (3c-1)$$

and then constant to last stage ( $i = j$  to  $n$ )

$$U_i = U_n \quad (3c-2)$$

Turbine specific work is

$$\overline{\Delta h'} = \frac{C_B}{J} \frac{P}{w} \quad (4)$$

and is equal to the sum of the specific work of the stages:

$$\overline{\Delta h'} = \sum_{i=1}^n \Delta h'_i \quad (5)$$

Expressing stage specific work in terms of stage work factor as

$$\Delta h'_i = \frac{U_i^2}{gJ} \psi \quad (6)$$

and substituting equation (6) into equation (5) yield

$$\overline{\Delta h'} = \sum_{i=1}^n \frac{U_i^2}{gJ} \psi \quad (7)$$

Since the velocity diagram shape is specified to be the same for all stages, the stage work factor is the same for all stages and is computed by rearranging equation (7) as

$$\psi = \frac{gJ \overline{\Delta h'}}{\sum_{i=1}^n U_i^2} \quad (8)$$

The value of  $\psi$  is the primary factor determining turbine efficiency.

The method used for computing turbine efficiency is basically similar to that presented in reference 2, but has the following

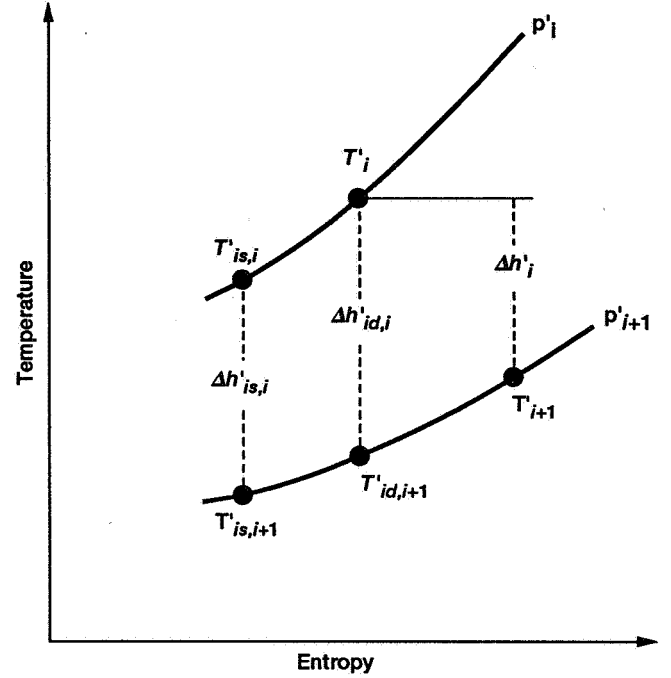


Figure 2.—Temperature-entropy diagram for  $i^{\text{th}}$  stage.

additional features: (1) the turbines considered in this report are not restricted to a constant mean-section diameter, (2) exit vanes to provide axial flow leaving the turbine can be included in the design, (3) the velocity diagrams also can be specified as symmetrical or by an input swirl split, and (4) the effect of turbine reheat is included.

Referring to figure 2, turbine overall total efficiency is defined as

$$\bar{\eta}' = \frac{\overline{\Delta h'}}{\Delta h'_{id}} = \frac{\overline{\Delta h'}}{\sum_{i=1}^n \Delta h'_{is,i}} = \frac{\overline{\Delta h'}}{\sum_{i=1}^n \left( \frac{T'_{is,i}}{T'_i} \right) \Delta h'_{id,i}} \quad (9a)$$

or alternately expressed as

$$\bar{\eta}' = \frac{1}{\sum_{i=1}^n \left( \frac{T'_{is,i}}{T'_i} \right) \left( \frac{\Delta h'_{id,i}}{\Delta h'_i} \right) \left( \frac{\Delta h'_i}{\Delta h'_{id,i}} \right)} \quad (9b)$$

Combining equations (6) and (7) yields

$$\frac{\Delta h'_i}{\overline{\Delta h'}} = \frac{U_i^2}{\sum_{i=1}^n U_i^2} \quad (10)$$

and, by definition

$$\eta'_i = \frac{\Delta h'_i}{\Delta h'_{id,i}} \quad (11)$$

Substituting equations (10) and (11) into equation (9b) yields

$$\bar{\eta}' = \frac{1}{\sum_1^n \left( \frac{T'_{is,i}}{T'_i} \right) \left( \frac{1}{\eta'_i} \right) \left( \frac{U_i^2}{\sum_1^n U_i^2} \right)} \quad (12)$$

The terms  $T'_i$  and  $T'_{is,i}$ , using equation (6), can be written as

$$T'_i = T_{i-1} - \frac{\Delta h'_{i-1}}{c_p} = T_{i-1} - \frac{U_{i-1}^2}{gJc_p} \psi \quad (13)$$

$$T'_{is,i} = T'_{is,i-1} - \frac{\Delta h'_{is,i-1}}{c_p} = T'_{is,i-1} - \left( \frac{T'_{is,i-1}}{T'_{i-1}} \right) \left( \frac{1}{\eta'_{i-1}} \right) \frac{U_{i-1}^2}{gJc_p} \psi \quad (14)$$

where

$$c_p = \frac{\gamma}{\gamma - 1} \frac{R}{J} \quad (15)$$

Since both  $T'_i$  and  $T'_{is,i}$  are equal to  $T'_{in}$  at the turbine inlet (i.e., at  $i = 1$ ), equations (13) and (14) can be evaluated by recursion once the  $\eta'_i$  are known.

The stage efficiency computation method is explained fully in reference 2, and only the key equations are presented here. Stage total efficiency can be expressed as

$$\eta' = \frac{1}{1 + \frac{A}{2} \psi} \quad (16)$$

where the stage loss parameter  $A$  is

$$A = \frac{K \text{Re}^{-0.2}}{\cot \alpha_1} (F_{st} C_{st} + F_{ro} C_{ro} + C_{ev}) \quad (17)$$

The constant of proportionality  $K$ , called the turbine loss coefficient in this report, must be determined empirically, and recommended values based on experimental efficiencies are presented later in this report. The Reynolds number used in this calculation is defined as

$$\text{Re} = \frac{2w}{\mu D_{m,in}} \quad (18)$$

Some of the terms within the parentheses in equation (17) are the same for all cases, while others depend on stage location, velocity diagram type, and use of exit vanes. The rotor weighting factor  $F_{ro}$  and rotor loss parameter  $C_{ro}$  are the same for all cases:

$$F_{ro} = 2 \quad (19)$$

$$C_{ro} = 2 \cot^2 \alpha_1 \left( \frac{V_{u,1}}{\Delta V_u} \right)^2 + \left( \frac{V_{u,1}}{\Delta V_u} - \frac{1}{\psi} \right)^2 + \left( \frac{V_{u,2}}{\Delta V_u} - \frac{1}{\psi} \right)^2 \quad (20)$$

For last stages of turbines having exit vanes,

$$C_{ev} = 2 \cot^2 \alpha_1 \left( \frac{V_{u,1}}{\Delta V_u} \right)^2 + \left( \frac{V_{u,2}}{\Delta V_u} \right)^2 \quad (21)$$

For all other stages

$$C_{ev} = 0 \quad (22)$$

For first-stage stators having a specified inlet angle  $\alpha_0$ , the stator loss parameter is expressed

$$C_{st} = \left[ 1 + \cot^2 \alpha_1 (2 + \tan^2 \alpha_0) \right] \left( \frac{V_{u,1}}{\Delta V_u} \right)^2 \quad (23)$$

For all other stators

$$C_{st} = (1 + 2 \cot^2 \alpha_1) \left( \frac{V_{u,1}}{\Delta V_u} \right)^2 + \left( \frac{V_{u,2}}{\Delta V_u} \right)^2 \quad (24)$$

The stator weighting factor  $F_{st}$  also depends on whether or not the stator is a first-stage stator. For a first-stage stator having a specified inlet angle  $\alpha_0$ ,

$$F_{st} = \frac{1 - \frac{3 \tan \alpha_0}{\tan \alpha_1}}{1 - \frac{\tan \alpha_0}{\tan \alpha_1}} \quad (25)$$

For all other stators

$$F_{st} = \frac{1 - \frac{3(V_{u,2}/\Delta V_u)}{V_{u,1}/\Delta V_u}}{1 - \frac{V_{u,2}/\Delta V_u}{V_{u,1}/\Delta V_u}} \quad (26)$$

The rotor inlet and exit swirl parameters  $V_{u,1}/\Delta V_u$  and  $V_{u,2}/\Delta V_u$  of equations (19) to (26) depend on the specific type of velocity diagram or are input for the general diagram. The following table presents the relations for evaluating the swirl parameters:

Velocity	Inlet swirl parameter, $V_{u,1}/\Delta V_u$	Exit swirl parameter, $V_{u,2}/\Delta V_u$
Symmetrical	$\frac{1+\psi}{2\psi}$	$\frac{1-\psi}{2\psi}$
Zero exit swirl	1	0
Impulse	$\frac{2+\psi}{2\psi}$	$\frac{2-\psi}{2\psi}$
General	Input	$\frac{V_{u,1}}{\Delta V_u} - 1$

With turbine total efficiency obtained from the foregoing equations, the turbine exit velocities and state conditions are computed as follows:

$$\Delta V_{u,n} = \psi U_n \quad (27)$$

$$V_{u,1,n} = \frac{V_{u,1}}{\Delta V_u} \Delta V_{u,n} \quad (28)$$

$$V_{u,2,n} = \frac{V_{u,2}}{\Delta V_u} \Delta V_{u,n} \quad (29)$$

$$V_{me,n} = V_{u,1,n} \cot \alpha_1 \quad (30)$$

$$V_{me,ex} = \sqrt{E} V_{me,n} \quad (31)$$

$$V_{u,ex} = \begin{cases} V_{u,2,n} & \text{no exit vanes} \\ 0 & \text{exit vanes} \end{cases} \quad (32)$$

$$V_{ex} = \sqrt{V_{me,ex}^2 + V_{u,ex}^2} \quad (33)$$

$$p'_{ex} = p'_{in} \left( 1 - \frac{\overline{\Delta h'}}{c_p T'_{in} \bar{\eta}'} \right)^{\gamma/(\gamma-1)} \quad (34)$$

$$T'_{ex} = T'_{in} - \frac{\overline{\Delta h'}}{c_p} \quad (35)$$

$$T_{ex} = T'_{ex} - \frac{V_{ex}^2}{2gJc_p} \quad (36)$$

$$p_{ex} = p'_{ex} \left( \frac{T_{ex}}{T'_{ex}} \right)^{\gamma/(\gamma-1)} \quad (37)$$

In order that the turbine total and static efficiencies be consistent with the computed exit velocity, the static efficiency is computed as

$$\bar{\eta} = \frac{\overline{\Delta h'}}{c_p T'_{in} \left[ 1 - \left( \frac{p_{ex}}{p'_{in}} \right)^{(\gamma-1)/\gamma} \right]} \quad (38)$$

In order to calculate continuity for a flowpath having significant inclination, it is necessary to determine a flowpath slope. Consistent with the preliminary nature of this analysis, a correlation relating axial chord to mean diameter was used to determine axial length. The correlation, based on the geometry of existing turbines, is of the form

$$c_x = k_1 D_m + k_2 D_m^2 + k_3 + k_4 D_m^{-1} \quad (39)$$

and was established over a range of mean diameters from 0.10 to 1.25 m (0.33 to 4.17 ft). Because of the scatter in the available data, correlations were determined for low, mean, and high aspect ratios. The correlation coefficients are presented in table I. The three correlation curves for ratio of axial chord to mean diameter are plotted in figure 3. Using one-third of axial chord length for spacing between blade rows, the slope for a linear meanline between stage 1 exit and stage  $n$  exit is

$$\tan \theta = \frac{0.5(D_{m,ex} - D_{m,in})}{2(4/3) \sum_2^n c_{x,i}} = \frac{3(D_{m,ex} - D_{m,in})}{16 \sum_2^n c_{x,i}} \quad (40)$$

For two-segment meanlines, the lower or upper limit for the summation is changed as appropriate for the linear section, and the slope for the constant section is set equal to zero.



TABLE I.—AXIAL CHORD CORRELATION COEFFICIENTS

$$|c_x = k_1 D_m + k_2 D_m^2 + k_3 + k_4 D_m^{-1}|$$

Aspect ratio	$k_1 \times 10^2$	$k_2 \times 10^3$		$k_3 \times 10^2$		$k_4 \times 10^3$	
		m <sup>-1</sup>	ft <sup>-1</sup>	m	ft	m <sup>2</sup>	ft <sup>2</sup>
High	0.5716	7.8984	2.4074	1.4911	4.8921	-0.7498	-8.0708
Mid	3.0740	9.9323	3.0274	1.4714	4.8275	-.7411	-7.9771
Low	6.6559	8.7134	2.6558	1.1949	3.9203	-.5580	-6.0063

Exit annulus area, radius ratio, and hub and tip diameters are obtained as follows:

$$\rho_{ex} = \frac{p_{ex}}{RT_{ex}} \quad (41)$$

$$V_{x,ex} = V_{me,ex} \cos \theta_n \quad (42)$$

$$A_{an,ex} = \frac{w}{\rho_{ex} V_{x,ex}} \quad (43)$$

$$\left(\frac{r_h}{r_t}\right)_{ex} = \frac{1 - \frac{A_{an,ex}}{\pi D_{m,ex}^2}}{1 + \frac{A_{an,ex}}{\pi D_{m,ex}^2}} \quad (44)$$

$$D_{t,ex} = \frac{2D_{m,ex}}{1 + \left(\frac{r_h}{r_t}\right)_{ex}} \quad (45)$$

$$D_{h,ex} = D_{t,ex} \left(\frac{r_h}{r_t}\right)_{ex} \quad (46)$$

Absolute and relative flow angles, which are the same for each stage, are computed from the last-stage velocities:

$$\alpha_2 = \tan^{-1} \frac{V_{u,2,n}}{V_{me,n}} \quad (47)$$

$$W_{u,1,n} = V_{u,1,n} - U_n \quad (48)$$

$$W_{u,2,n} = V_{u,2,n} - U_n \quad (49)$$

$$\beta_1 = \tan^{-1} \frac{W_{u,1,n}}{V_{me,n}} \quad (50)$$

$$\beta_2 = \tan^{-1} \frac{W_{u,2,n}}{V_{me,n}} \quad (51)$$

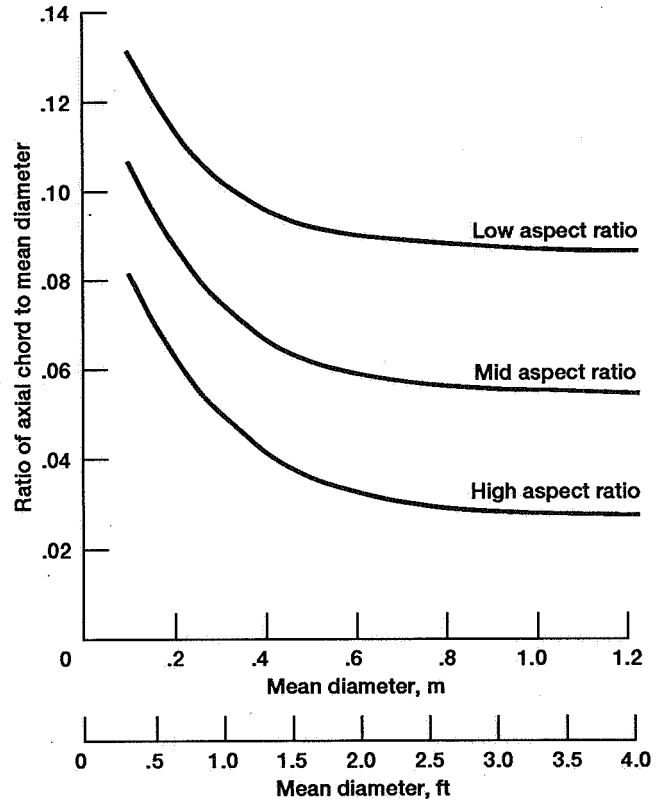


Figure 3.—Axial chord correlation.

Absolute and relative Mach numbers are computed for the last stage, where velocities are highest and temperatures are lowest, thus making the Mach numbers most severe:

$$V_{2,n} = \sqrt{V_{u,2,n}^2 + V_{me,ex}^2} \quad (52)$$

$$W_{2,n} = \sqrt{W_{u,2,n}^2 + V_{me,ex}^2} \quad (53)$$

$$W_{1,n} = \sqrt{W_{u,1,n}^2 + V_{me,n}^2} \quad (54)$$

$$V_{1,n} = \sqrt{V_{u,1,n}^2 + V_{me,n}^2} \quad (55)$$

$$T'_{1,n} = T'_{2,n} = T'_{ex} - \frac{V_{2,n}^2 - W_{2,n}^2}{2gJc_p} \quad (56)$$

$$T_{1,n} = T'_{1,n} - \frac{W_{1,n}^2}{2gJc_p} \quad (57)$$

$$M_{abs,2} = \frac{V_{2,n}}{\sqrt{\gamma g R T'_{ex}}} \quad (58)$$

$$M_{rel,2} = \frac{W_{1,n}}{\sqrt{\gamma g R T'_{ex}}} \quad (59)$$

$$M_{rel,1} = \frac{W_{1,n}}{\sqrt{\gamma g R T_{1,n}}} \quad (60)$$

$$M_{abs,1} = \frac{V_{1,n}}{\sqrt{\gamma g R T_{1,n}}} \quad (61)$$

Efficiency and continuity computations are based solely on the meanline flow parameters. Although radial variations of velocity in a turbine need not be of the free-vortex type, the free-vortex hub and tip velocity-diagram parameters can give a qualitative indication of the possible severity of end wall flow conditions. With blade speed directly proportional to diameter

$$U \propto D \quad (62)$$

and applying the free-vortex conditions for radial variations of the velocity components

$$V_u \propto \frac{1}{D} \quad (63)$$

and

$$V_{me} = \text{Constant} \quad (64)$$

the hub and tip velocity diagrams are determined from equations (47) to (61).

In order to establish the flow annulus geometry near the turbine inlet, a flow analysis is made at the first-stage exit as follows:

$$\Delta h'_a = \frac{U_a^2}{gJ} \psi \quad (65)$$

$$T'_{2,a} = T'_{in} - \frac{\Delta h'_a}{c_p} \quad (66)$$

$$p'_{2,a} = p'_{in} \left( 1 - \frac{\Delta h'_a}{c_p T'_{in} \eta'_a} \right)^{\gamma/(\gamma-1)} \quad (67)$$

$$\Delta V_{u,a} = \psi U_a \quad (68)$$

$$V_{u,2,a} = \frac{V_{u,2}}{\Delta V_u} \Delta V_{u,a} \quad (69)$$

$$V_{me,2,a} = \sqrt{E} \frac{V_{u,1}}{\Delta V_u} \Delta V_{u,a} \cot \alpha_1 \quad (70)$$

$$V_{2,a} = \sqrt{V_{u,2,a}^2 + V_{me,2,a}^2} \quad (71)$$

$$T_{2,a} = T'_{2,a} - \frac{V_{2,a}^2}{2gJc_p} \quad (72)$$

$$p_{2,a} = p'_{2,a} \left( \frac{T_{2,a}}{T'_{2,a}} \right)^{\gamma/(\gamma-1)} \quad (73)$$

$$V_{x,2,a} = V_{me,2,a} \cos \theta_a \quad (74)$$

$$\rho_{2,a} = \frac{p_{2,a}}{RT_{2,a}} \quad (75)$$

$$A_{an,2,a} = \frac{w}{\rho_{2,a} V_{x,2,a}} \quad (76)$$

$$\left( \frac{r_h}{r_t} \right)_{2,a} = \frac{1 - \frac{A_{an,2,a}}{\pi D_{m,in}^2}}{1 + \frac{A_{an,2,a}}{\pi D_{m,in}^2}} \quad (77)$$

$$D_{t,2,a} = \frac{2D_{m,in}}{1 + \left( \frac{r_h}{r_t} \right)_{2,a}} \quad (78)$$

$$D_{h,2,a} = D_{t,2,a} \left( \frac{r_h}{r_t} \right)_{2,a} \quad (79)$$

When a constant annulus is assumed for the first stage, the first-stage exit dimensions become the inlet dimensions for that turbine.

### Alternative Calculation Procedure

The basic calculation procedure described in the previous section requires as inputs the inlet and exit mean diameters, stator exit angle, and shaft power. Alternatively, the hub or tip diameters could be specified as input, and the mean diameters computed; the exit radius ratio could be specified as input, and the stator exit angle computed; and/or the turbine pressure ratio could be specified as input, and the shaft power computed. These alternative input options require iterative calculation procedures such as described in this section.

With hub or tip diameters rather than mean diameters specified at the inlet and exit, it is necessary to assume initial values for the inlet and exit radius ratios. Initial values for inlet and exit mean diameters are then obtained as

$$D_m = \frac{\left(1 + \frac{r_h}{r_t}\right) D_h}{2 \frac{r_h}{r_t}} \quad (80)$$

when hub diameter is input and

$$D_m = \frac{\left(1 + \frac{r_h}{r_t}\right) D_t}{2} \quad (81)$$

when tip diameter is input. The computation then proceeds from equation (1) to equation (44), and the computed exit radius ratio is compared with the assumed value. If they are not the same (within a given tolerance), then the computed value of exit radius ratio is used to calculate a new value for exit mean diameter (from eq. (80) or (81)), and the computation procedure is repeated until convergence is obtained. Then, computation proceeds through equation (77), and the computed inlet radius ratio is compared with the assumed value. If they are not within tolerance, the computed value of inlet radius ratio is used to calculate a new value for inlet mean diameter, and the computation procedure is repeated from equation (1). This entire procedure is repeated until both inlet and exit radius ratios converge in the same calculation pass. When hub or tip diameter rather than mean diameter is used as input, a two-segment meanline cannot be specified.

With exit radius ratio rather than stator exit angle specified, a value of stator exit angle is assumed for the evaluation of equation (17). The computation proceeds through equation (41). Equation (44) is then used to compute the exit annulus area from the input value of radius ratio, and the exit axial velocity is then obtained from equation (43). The density used in equation (43), however, is not consistent with the exit area, and equations (43), (42), (33), (36), and (41) must be iterated until convergence is obtained. Then, the stator exit angle is computed as

$$\alpha_1 = \tan^{-1} \frac{V_{u,1,n}}{V_{me,n}} \quad (82)$$

and compared with the assumed value. If they are not within tolerance, the computed value of stator exit angle is used for

the evaluation of equation (17), and the computation procedure just given is repeated until stator exit angle converges. The remainder of the computation is then completed.

With the turbine inlet-total- to exit-static-pressure ratio rather than shaft power specified as input, an initial value of turbine static efficiency is assumed. Turbine work is then computed from

$$\overline{\Delta h'} = \bar{\eta} c_p T'_{in} \left[ 1 - \left( \frac{p_{ex}}{p'_{in}} \right)^{(\gamma-1)/\gamma} \right] \quad (83)$$

instead of from equation (4). The computation then proceeds through equation (38) to compute a static efficiency. If the computed and the assumed values are not within tolerance, a new value of static efficiency is assumed, and the computation is repeated until convergence. The remainder of the computation is then completed.

## Loss-Coefficient Evaluation

Ten turbines that were designed for airbreathing engine applications and performance tested in component facilities were selected to serve as the basis for the loss coefficient evaluation. These turbines represent the output of several different design systems and cover a wide range of design characteristics. Six of the turbines were designed and tested by NASA, two were designed and tested by GE Aircraft Engines Co., and two were designed by Pratt & Whitney, one of which was tested by Pratt & Whitney and the other by NASA. The characteristics of these turbines are presented in table II. The number of stages varies from 1 to 5, stage work factors ( $\psi$ ) from 1.2 to 4.7, stator angles from 51° to 80°, exit mean diameters, except for one, from 47 to 69 cm (18 to 27 in.), and radius ratios from 0.50 to 0.86. All the listed total efficiencies except for two were as measured. In those two cases, corrections were made to account for the effects of cooling air (ref. 4) and a large discrepancy in flow rate from design intent (ref. 5).

These turbines were modeled as closely as possible for the subject computer program, and design performance was computed over a range of loss coefficients. The variation of efficiency with loss coefficient was a direct proportion between the quantity  $(1 - \eta)$  and loss coefficient,  $K$ . Calculated efficiencies are plotted against measured efficiencies in figure 4 for a constant loss coefficient of 0.3. As seen, for all cases except one, the calculated efficiency was within one point of the measured efficiency. In view of the simplistic modeling used for this computer program, this agreement is admittedly fortuitous. It does illustrate, however, that this program is capable of predicting reasonable efficiencies over a wide range of airbreathing engine turbine conditions. A loss coefficient value of 0.3 is recommended for use in the absence of additional information.

TABLE II.—TURBINES USED FOR EVALUATION

Number of stages	Average stage work factor, $\psi$	Average stator angle, deg	Exit mean diameter		Exit radius ratio	Measured efficiency	Reference
			cm	in.			
1	1.7	67.0	66.0	26.0	0.73	0.923	6
2	1.7	64.5	66.0	26.0	.63	.932	7
1	1.2	63.4	20.3	8.0	.68	.917	8
1	1.9	73.1	47.0	18.5	.85	.886	9
3.5	4.0	51.4	49.8	19.6	.50	.880	5
4.5	4.7	66.6	49.8	19.6	.57	.855	10
1	1.7	75.0	46.7	18.4	.86	.895	11
1	1.6	79.6	56.4	22.2	.85	.901	12
2	1.3	71.6	69.3	27.3	.82	.898	4
5	2.6	61.0	63.5	25.0	.62	.920	13

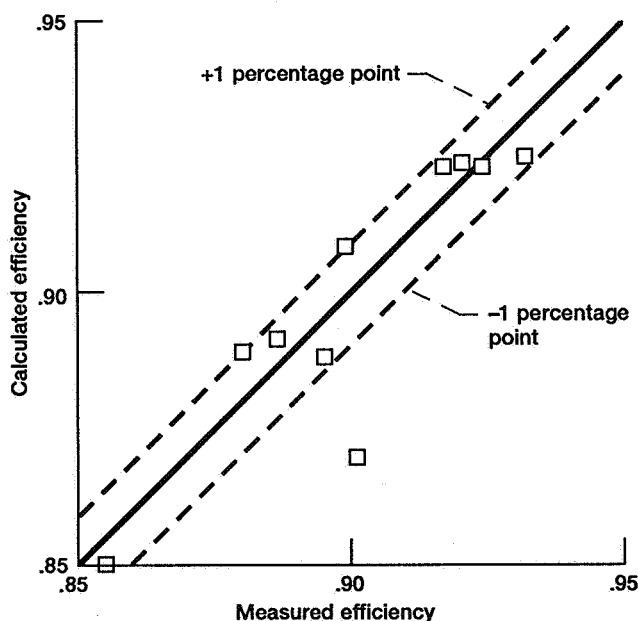


Figure 4.—Comparison of measured and calculated total efficiencies. Loss coefficient, 0.3.

## Description of Input and Output

This section presents a detailed description of the program input, normal output, and error messages. Included in the input and output sections are several example cases illustrating the use of the program and the various options.

### Input

The program input, a sample of which is presented in table III, consists of a title record and the required physical data and option indicators in NAMELIST form. The title, which is printed as a heading on the output listing, can contain up to 77 characters located anywhere in columns 2 to 78 on the title record. A title, even if it is left blank, must be the first record of the input data. Additional titles can be used to identify different cases being run in the same data file. This is done by placing a title in front of the data for the particular case and using the option indicator ITIT as subsequently described.

The physical data and option indicators are input in data records having the NAMELIST name INPUT. The variables and indicators that compose INPUT and the proper units are as follows. These must be input for all cases except where otherwise indicated. Either the SI units or the U.S. customary units shown below may be used.

PTIN	inlet total pressure, N/cm <sup>2</sup> ; lb/in. <sup>2</sup>
TTIN	inlet total temperature, K; °R
MU	gas viscosity, N·sec/m <sup>2</sup> ; lb/sec·ft
R	gas constant, J/kg·K; ft·lbf/lbm·°R
GAM	specific heat ratio
DIN	inlet diameter—hub or mean or tip value as specified by the indicator IDIAM, cm; in.
DEX	exit diameter—hub or mean or tip value as specified by the indicator IDIAM, cm; in.
RREX	exit radius ratio; RREX may be omitted in the case where both IDIAM = 2 and IALPH = 0; RREX is used as first trial value when IALPH = 0 and IDIAM = 1 or 3
RPM	rotative speed, rad/sec; rpm
POW	shaft power—omit when IPR = 1, kW; hp
W	mass flow rate, kg/sec; lb/sec

TABLE III.—SAMPLE INPUT

```

THESE ARE SAMPLE CASES FOR THIS PROGRAM. U.S. CUSTOMARY UNIT ARE USED.
&INPUT PTIN=113.1,TTIN=2660.,MU=.376E-04,R=53.37,GAM=1.302,DIN=22.,DEX=24.,
RPM=11400.,POW=12900.,W=53.5,ALPHA=65.,KLOSS=.35,NMIN=2,NMAX=2,E=1.2,
IALPH=0,IDIAM=2,IVD=1,IEV=0,IPR=0,IU=2 &END
&INPUT IDIAM=1,RREX=.8,DIN=20.,DEX=21.,ITIT=1 &END
THIS IS AN ADDITIONAL TITLE CALLED FOR BY ITIT IN PREVIOUS CASE DATA
&INPUT IALPH=1,IEV=1, &END
&INPUT IALPH=0,IEV=0,IVD=2 &END
&INPUT IVD=3 &END
&INPUT IPR=1,IVD=1,PRTS=3.66 &END

```

ALPHA stator exit angle from axial direction; ALPHA is used as first trial value when IALPH = 1, deg

ALPHA0 turbine inlet flow angle; required only when KALPH0 = 2, deg

VUIDVU ratio of rotor inlet swirl to total change in swirl; input only when IVD = 5

KLOSS turbine loss coefficient; a value of 0.3 is recommended in the absence of additional information

NMIN minimum number of stages for which the calculations are performed

NMAX maximum number of stages for which the calculations are performed; results are obtained for all stage numbers between NMIN and NMAX

NMID stage number at which meanline changes slope; may be omitted when IMID = 0

E squared ratio of stage-exit to stage-average meridional velocities

PRTS turbine inlet-total to exit-static-pressure ratio; omit when IPR = 0

IALPH indicates whether stator exit angle or turbine exit radius ratio is specified:  
IALPH = 0—turbine is designed for specified ALPHA  
IALPH = 1—turbine is designed for specified RREX

IDIAM indicates whether input diameters are hub, mean, or tip values:  
IDIAM = 1—input diameters are hub values  
IDIAM = 2—input diameters are mean values  
IDIAM = 3—input diameters are tip values

IVD indicates type of velocity diagram used:  
IVD = 1—symmetrical diagrams  
IVD = 2—zero exit swirl diagrams  
IVD = 3—impulse diagrams  
IVD = 4—zero exit swirl diagrams if  $\psi \leq 2.0$  and impulse diagrams if  $\psi \geq 2.0$   
IVD = 5—ratio of rotor inlet swirl to total change in swirl is input as VUIDVU

ITIT indicates use of titles in addition to that required as first line of data package:  
ITIT = 1—title line precedes next data set; must be input for each additional title because ITIT is automatically restored to zero after each title is read

IEV indicates use of exit vanes:  
IEV = 0—no exit vanes  
IEV = 1—exit vanes are used to turn turbine exit flow to axial direction

IPR indicates whether shaft power or pressure ratio is specified:  
IPR = 0—shaft power is input  
IPR = 1—turbine inlet-total- to exit-static-pressure ratio is input

IU indicates type of units used for input and output:  
IU = 1—SI units  
IU = 2—U.S. customary units

KALPH0 indicates turbine-inlet flow angle option:  
KALPH0 = 0—turbine-inlet flow is axial (default)  
KALPH0 = 1—turbine-inlet flow angle equals stage-exit flow angle  
KALPH0 = 2—turbine-inlet flow angle is input as ALPHA0

IAR indicates blading aspect ratio:  
IAR = 1—high aspect-ratio blading  
IAR = 2—mid aspect-ratio blading (default)  
IAR = 3—low aspect-ratio blading

IMID indicates meanline shape:  
IMID = 0—meanline linear from stage 1 to stage  $N$  (default)  
IMID = 1—meanline constant from stage 1 to stage NMID and then linear to stage  $N$   
IMID = 2—meanline linear from stage 1 to stage NMID and then constant to stage  $N$

The first line of the input file shown in table III is the mandatory title card, which can contain any desired message. The next three lines are the first data set, which contains all required inputs. This first case represents computation in accordance with the basic calculation procedure described previously. Data input for subsequent cases need only include those values that differ from previous case data. The fifth line is the second data set and represents the option where hub diameter is input. Also, the second case data specify that a title, which is the sixth data line, precede the third case data. Cards 7 to 10 represent four additional cases illustrating the use of different input options. The output corresponding to this sample input is described in the following section.

### Output

The program output consists of title headings, the input variables, and computed results. This section presents normal output. Error message output is described in the next section.

Table IV presents the output that corresponds to the sample input shown in table III. The top line of output is a program identification title that is automatically printed. The second line is the title card message. The next four lines are the input variables and their associated values for the first data set. The input variable names are spelled out. The units for the input variable values are as described in the "Input" section. The zeros

printed under EXIT RADIUS RATIO and T-S PRESSURE RATIO indicate that these are computed for this case and not specified by the input. The fact that the input diameters are mean diameters is indicated by the MN in the variable name. Hub and tip diameters would be indicated by HB and TP, respectively. These four lines of output are printed for each new data set.

The next group of eight lines is the computation results satisfying the input requirements. The output parameters are spelled out and are self-explanatory. These temperatures, pressures, velocities, and angles are meanline values. On the first line are the number of stages, the stage work factor, the Reynolds number, and the diagram type, which is symmetrical for this first case. The remainder of the output includes exit and inlet tip and hub diameters in the first column, exit total and static temperatures and pressures in the second column, total and static efficiencies and velocity diagram angles in the third column, and first- and last-stage blade speeds and last-stage absolute velocity components in the last column. The last line of this output group presents the last-stage absolute and relative Mach numbers.

The next group of four lines is the hub and tip free-vortex values of Mach numbers and angles. The four columns present the stator-exit absolute, rotor-inlet relative, rotor-exit relative, and stage-exit absolute values. The last line of output for each case is the meanline slope based on the specified (mid in this case) aspect-ratio blading.

After the computations for each input case are completed, the input data for the next case are printed. The second case presented here is that where the specified diameters are hub values. The third input case in table IV is preceded by an additional title line. This causes the next output to begin at the top of a new page with the program identification and the title message.

The third case is for a specified exit radius ratio rather than for a specified stator exit angle. This is indicated by a zero appearing under STATOR EX ANG in the row of input variables. Exit vanes are included in this case, as indicated by WITH EXIT VANES printed after number of stages on the first line of result output. The fourth and fifth cases, as indicated by the top line of the fourth column of result output, are for zero exit swirl and impulse diagrams, respectively. Turbine pressure ratio rather than shaft power is specified in the sixth case. For this case, the input pressure ratio was chosen to be the same as the computed value obtained for the second case. As should be expected, the program converges to identical solutions for both cases. An extra line of output consisting of shaft power and specific work is printed for the case where pressure ratio is specified as input.

### Error Messages

The program contains seven output messages. Five of these indicate the nonexistence of a meanline solution satisfying

TABLE IV.—SAMPLE OUTPUT

#### TURBINE VELOCITY DIAGRAM ANALYSIS

THESE ARE SAMPLE CASES FOR THIS PROGRAM. U.S. CUSTOMARY UNIT ARE USED.

SHAFT POWER	MASS FLOW	INLET TEMP	INLET PRESS	ROTATIVE SPEED	INLET MN DIA	EXIT MN DIA	EXIT RADIUS RATIO	STATOR EX ANG	GAS CONST	HEAT CAPAC RATIO	GAS VISCOSITY	TURBINE LOSS COEF	AXIAL VEL SQ RATIO	T-S PRES RATI					
12900.0	53.50	2660.00	113.10	11400.00	22.00	24.00	0.0000	65.00	53.37	1.302	0.376E-04	0.350	1.200	0.					
STAGES= 2																			
EXIT TIP DIAMETER = 27.18					STAGE WORK FACTOR= 1.63					REYNOLDS NO.=0.1552E+07					DIAGRAMS ARE SYMMETRICAL				
EXIT HUB DIAMETER = 20.82					EXIT TOTAL TEMP =2083.64					STATOR EXIT ANGLE= 65.00					FIRST STAGE MEAN SPEED= 1094.32				
EXIT RADIUS RATIO =0.7658					EXIT STATIC TEMP =2030.85					STAGE EXIT ANGLE =-27.10					LAST STAGE MEAN SPEED = 1193.81				
INLET TIP DIAMETER= 24.16					EXIT TOTAL PRESS = 34.85					ROTOR INLET ANGLE= 27.10					LAST STAGE INLET SWIRL= 1567.87				
INLET HUB DIAMETER= 19.84					EXIT STATIC PRESS= 31.20					ROTOR EXIT ANGLE =-65.00					LAST STAGE EXIT SWIRL = -374.06				
INLET RADIUS RATIO=0.8213					T-T PRESS RATIO = 3.245					TOTAL EFFICIENCY =0.907					LAST STAGE MERID VELOC= 800.93				
LAST STG M1 ABS =0.7810					T-S PRESS RATIO = 3.625					STATIC EFFICIENCY=0.839					EXIT MERID MACH NUMBER= 0.3759				
					LAST STG M1 REL = 0.3708					LAST STG M2 REL =0.8263					LAST STG M2 ABS = 0.4149				
HUB:	LAST STG M1 ABS =0.8915				LAST STG M1 REL = 0.4862				LAST STG M2 REL =0.7684				LAST STG M2 ABS = 0.3980						
TIP:	STATOR EXIT ANGLE = 67.98				ROTOR INLET ANGLE = 46.57				ROTOR EXIT ANGLE =-63.51				STAGE EXIT ANGLE =-30.53						
	LAST STG M1 ABS =0.7010				LAST STG M1 REL = 0.3277				LAST STG M2 REL =0.8590				LAST STG M2 ABS = 0.3757						
	STATOR EXIT ANGLE = 62.16				ROTOR INLET ANGLE = 2.51				ROTOR EXIT ANGLE =-66.51				STAGE EXIT ANGLE =-24.31						
STAGE 1- 2 MEANLINE SLOPE = 14.85 DEG BASED ON MID ASPECT-RATIO BLADING																			
SHAFT POWER	MASS FLOW	INLET TEMP	INLET PRESS	ROTATIVE SPEED	INLET HB DIA	EXIT HB DIA	EXIT RADIUS RATIO	STATOR EX ANG	GAS CONST	HEAT CAPAC RATIO	GAS VISCOSITY	TURBINE LOSS COEF	AXIAL VEL SQ RATIO	T-S PRES RATI					
12900.0	53.50	2660.00	113.10	11400.00	20.00	21.00	0.0000	65.00	53.37	1.302	0.376E-04	0.350	1.200	0.					
STAGES= 2																			
EXIT TIP DIAMETER = 27.32					STAGE WORK FACTOR= 1.61					REYNOLDS NO.=0.1542E+07					DIAGRAMS ARE SYMMETRICAL				
EXIT HUB DIAMETER = 21.00					EXIT TOTAL TEMP =2083.64					STATOR EXIT ANGLE= 65.00					FIRST STAGE MEAN SPEED= 1101.56				
EXIT RADIUS RATIO =0.7685					EXIT STATIC TEMP =2031.50					STAGE EXIT ANGLE =-26.48					LAST STAGE MEAN SPEED = 1201.87				
INLET TIP DIAMETER= 24.29					EXIT TOTAL PRESS = 34.87					ROTOR INLET ANGLE= 26.48					LAST STAGE INLET SWIRL = 1565.51				
INLET HUB DIAMETER= 20.00					EXIT STATIC PRESS= 31.27					ROTOR EXIT ANGLE =-65.00					LAST STAGE EXIT SWIRL = -363.64				
INLET RADIUS RATIO=0.8233					T-T PRESS RATIO = 3.243					TOTAL EFFICIENCY =0.907					LAST STAGE MERID VELOC= 799.73				
LAST STG M1 ABS =0.7798					T-S PRESS RATIO = 3.617					STATIC EFFICIENCY=0.840					EXIT MERID MACH NUMBER= 0.3753				
					LAST STG M1 REL = 0.3682					LAST STG M2 REL =0.8249					LAST STG M2 ABS = 0.4123				
HUB:	LAST STG M1 ABS =0.8883				LAST STG M1 REL = 0.4805				LAST STG M2 REL =0.7664				LAST STG M2 ABS = 0.3944						
TIP:	STATOR EXIT ANGLE = 67.94				ROTOR INLET ANGLE = 46.03				ROTOR EXIT ANGLE =-63.48				STAGE EXIT ANGLE =-29.82						
	LAST STG M1 ABS =0.7007				LAST STG M1 REL = 0.3271				LAST STG M2 REL =0.8580				LAST STG M2 ABS = 0.3735						
	STATOR EXIT ANGLE = 62.20				ROTOR INLET ANGLE = 1.97				ROTOR EXIT ANGLE =-66.52				STAGE EXIT ANGLE =-23.77						
STAGE 1- 2 MEANLINE SLOPE = 14.89 DEG BASED ON MID ASPECT-RATIO BLADING																			

TABLE IV.—Concluded.

### TURBINE VELOCITY DIAGRAM ANALYSIS

THIS IS AN ADDITIONAL TITLE CALLED FOR BY ITIT IN PREVIOUS CASE DATA

SHAFT POWER	MASS FLOW	INLET TEMP	INLET PRESS	ROTATIVE SPEED	INLET HB DIA	EXIT HB DIA	EXIT RADIUS RATIO	STATOR EX ANG	GAS CONST	HEAT CAPAC RATIO	GAS VISCOSITY	TURBINE LOSS COEF	AXIAL VEL SQ RATIO	T-S PRESS RATIO
12900.0	53.50	2660.00	113.10	11400.00	20.00	21.00	0.8000	0.00	53.37	1.502	0.376E-04	0.350	1.200	0.000
STAGES= 2 W/EXT VANES      STAGE WORK FACTOR= 1.67      REYNOLDS NO.=0.1569E+07      DIAGRAMS ARE SYMMETRICAL														
EXIT TIP DIAMETER = 26.25			EXIT TOTAL TEMP =2083.64			STATOR EXIT ANGLE= 59.54			FIRST STAGE MEAN SPEED= 1082.83					
EXIT HUB DIAMETER = 21.00			EXIT STATIC TEMP =2014.99			STAGE EXIT ANGLE =-23.12			LAST STAGE MEAN SPEED= 1175.15					
EXIT RADIUS RATIO =0.8000			EXIT TOTAL PRESS = 34.80			ROTOR INLET ANGLE= 23.12			LAST STAGE INLET SWRTL= 1569.27					
INLET TIP DIAMETER= 23.54			EXIT STATIC PRESS= 30.10			ROTOR EXIT ANGLE =-59.54			LAST STAGE EXIT SWIRL = -394.12					
INLET HUB DIAMETER= 20.00			T-T PRESS RATIO = 3.250			TOTAL EFFICIENCY=0.906			LAST STAGE MERID VELOC= 1011.01					
INLET RADIUS RATIO=0.8497			T-S PRESS RATIO = 3.758			STATIC EFFICIENCY=0.819			EXIT MERID MACH NUMBER= 0.4764					
LAST STG M1 ABS =0.8263			LAST STG M1 REL = 0.4555			LAST STG M2 REL =0.8797			LAST STG M2 ABS = 0.5113					
HUB:	STATOR STG M1 ABS =0.9135			LAST STG M1 REL = 0.5370			LAST STG M2 REL =0.8255			LAST STG M2 ABS = 0.4827				
	STATOR EXIT ANGLE = 62.40			ROTOR INLET ANGLE = 37.99			ROTOR EXIT ANGLE =-58.19			STAGE EXIT ANGLE = -25.66				
TIP:	LAST STG M1 ABS =0.7603			LAST STG M1 REL = 0.4186			LAST STG M2 REL =0.8945			LAST STG M2 ABS = 0.4656				
	STATOR EXIT ANGLE = 56.84			ROTOR INLET ANGLE = 6.59			ROTOR EXIT ANGLE =-60.93			STAGE EXIT ANGLE = -21.02				

STAGE 1- 2 MEANLINE SLOPE = 13.98 DEG BASED ON MID ASPECT-RATIO BLADING

SHAFT POWER	MASS FLOW	INLET TEMP	INLET PRESS	ROTATIVE SPEED	INLET HB DIA	EXIT HB DIA	EXIT RADIUS RATIO	STATOR EX ANG	GAS CONST	HEAT CAPAC RATIO	GAS VISCOSITY	TURBINE LOSS COEF	AXIAL VEL SQ RATIO	T-S PRESS RATIO
12900.0	53.50	2660.00	113.10	11400.00	20.00	21.00	0.0000	65.00	53.37	1.302	0.376E-04	0.350	1.200	0.000
STAGES= 2		STAGE WORK FACTOR= 1.67		REYNOLDS NO.=0.1568E+07		DIAGRAMS ARE ZERO EX SWRL								
EXIT TIP DIAMETER = 26.34		EXIT TOTAL TEMP =2083.64		STATOR EXIT ANGLE= 65.00		FIRST STAGE MEAN SPEED= 1083.42								
EXIT HUB DIAMETER = 21.00		EXIT STATIC TEMP =2015.77		STAGE EXIT ANGLE = 0.00		LAST STAGE MEAN SPEED = 1177.29								
EXIT RADIUS RATIO =0.7974		EXIT TOTAL PRESS = 34.43		ROTOR INLET ANGLE= 40.62		LAST STAGE INLET SWIRL= 1962.12								
INLET TIP DIAMETER= 23.56		EXIT STATIC PRESS= 29.85		ROTOR EXIT ANGLE =-52.15		LAST STAGE EXIT SWIRL = 0.00								
INLET HUB DIAMETER= 20.00		T-T PRESS RATIO = 3.285		TOTAL EFFICIENCY=0.899		LAST STAGE MERID VELOC= 1002.33								
INLET RADIUS RATIO=0.8488		T-S PRESS RATIO = 3.789		STATIC EFFICIENCY=0.815		EXIT MERID MACH NUMBER= 0.4722								
LAST STG M1 ABS =1.0042		LAST STG M1 REL = 0.5592		LAST STG M2 REL =0.7284		LAST STG M2 ABS = 0.4722								
HUB: LAST STG M1 ABS =1.1294		LAST STG M1 REL = 0.6998		LAST STG M2 REL =0.6523		LAST STG M2 ABS = 0.4298								
STATOR EXIT ANGLE = 67.52		ROTOR INLET ANGLE = 51.90		ROTOR EXIT ANGLE =-48.78		STAGE EXIT ANGLE = 0.00								
TIP: LAST STG M1 ABS =0.9106		LAST STG M1 REL = 0.4681		LAST STG M2 REL =0.7507		LAST STG M2 ABS = 0.4298								
STATOR EXIT ANGLE = 62.58		ROTOR INLET ANGLE = 26.36		ROTOR EXIT ANGLE =-55.07		STAGE EXIT ANGLE = 0.00								

STAGE 1- 2 MEANLINE SLOPE = 14.19 DEG BASED ON MID ASPECT-RATIO BLADING

SHAFT POWER	MASS FLOW	INLET TEMP	INLET PRESS	ROTATIVE SPEED	INLET HB DIA	EXIT HB DIA	EXIT RADIUS RATIO	STATOR EX ANG	GAS CONST	HEAT CAPAC RATIO	GAS VISCOSITY	TURBINE LOSS COEF	AXIAL VEL SQ RATIO	T-S PRESS RATIO
12900.0	53.50	2660.00	113.10	11400.00	20.00	21.00	0.0000	65.00	53.37	1.302	0.376E-04	0.350	1.200	0.000
STAGES= 2      STAGE WORK FACTOR= 1.68      REYNOLDS NO.=0.1576E+07      DIAGRAMS ARE IMPULSE EXIT TIP DIAMETER = 26.11      EXIT TOTAL TEMP =2085.64      STATOR EXIT ANGLE= 65.00      FIRST STAGE MEAN SPEED= 1078.13 EXIT HUB DIAMETER = 21.00      EXIT STATIC TEMP =1999.26      STAGE EXIT ANGLE = 10.46      LAST STAGE MEAN SPEED= 1171.57 EXIT RADIUS RATIO =0.8044      EXIT TOTAL PRESS = 33.80      ROTOR INLET ANGLE= 44.42      LAST STAGE INLET SWIRL= 2157.44 EXIT TIP DIAMETER= 23.35      EXIT STATIC PRESS= 28.28      ROTOR EXIT ANGLE =-44.42      LAST STAGE EXIT SWIRL = 185.70 INLET HUB DIAMETER= 20.00      T-T PRESS RATIO = 3.346      TOTAL EFFICIENCY =0.887      LAST STAGE MERID VELOC= 1102.11 INLET RADIUS RATIO=0.8566      T-S PRESS RATIO = 3.999      STATIC EFFICIENCY=0.788      EXIT MERID MACH NUMBER= 0.5213 LAST STG M1 ABS =1.1222      LAST STG M1 REL = 0.6640      LAST STG M2 REL =0.6995      LAST STG M2 ABS = 0.5287														
HUB:	LAST STG M1 ABS	=1.2610	LAST STG M1 REL	= 0.8199	LAST STG M2 REL	=0.6168	LAST STG M2 ABS	= 0.4844						
	STATOR EXIT ANGLE	= 67.42	ROTOR INLET ANGLE	= 53.81	ROTOR EXIT ANGLE	=-39.74	STAGE EXIT ANGLE	= 11.70						
TIP:	LAST STG M1 ABS	=1.0182	LAST STG M1 REL	= 0.5561	LAST STG M2 REL	= 0.7135	LAST STG M2 ABS	= 9.4808						
	STATOR EXIT ANGLE	= 62.67	ROTOR INLET ANGLE	= 32.78	ROTOR EXIT ANGLE	=-48.35	STAGE EXIT ANGLE	= 0.6085						

STAGE 1- 2 MEANLINE SLOPE = 14.18 DEG BASED ON MID ASPECT-RATIO BLADING

SHAFT POWER	MASS FLOW	INLET TEMP	INLET PRESS	ROTATIVE SPEED	INLET HB DIA	EXIT HB DIA	EXIT RADIUS RATIO	STATOR EX ANG	GAS CONST	HEAT CAPAC RATIO	GAS VISCOSITY	TURBINE LOSS COEF	AXIAL VEL SQ	T-S PRESS RATIO
0.0	53.50	2660.00	113.10	11400.00	20.00	21.00	0.0000	65.00	53.37	1.302	0.376E-04	0.350	1.200	3.660
STAGES= 2				STAGE WORK FACTOR= 1.62			REYNOLDS NO.=0.1542E+07				DIAGRAMS ARE SYMMETRICAL			
EXIT TIP DIAMETER = 27.35				EXIT TOTAL TEMP =2079.52			STATOR EXIT ANGLE= 65.00				FIRST STAGE MEAN SPEED= 1101.57			
EXIT HUB DIAMETER = 21.00				EXIT STATIC TEMP =2026.66			STAGE EXIT ANGLE =-26.78				LAST STAGE MEAN SPEED = 1202.59			
EXIT RADIUS RATIO = 0.7677				EXIT TOTAL PRESS = 34.53			ROTOR INLET ANGLE= 26.78				LAST STAGE INLET SWIRL= 1572.70			
INLET TIP DIAMETER= 24.29				EXIT STATIC PRESS= 30.90			ROTOR EXIT ANGLE =-65.00				LAST STAGE EXIT SWIRL = -370.11			
INLET HUB DIAMETER= 20.00				T-T PRESS RATIO = 3.276			TOTAL EFFICIENCY= 0.907				LAST STAGE MERID VELOC= 803.40			
INLET RADIUS RATIO= 0.8233				T-S PRESS RATIO = 3.660			STATIC EFFICIENCY=0.840				EXIT MERID MACH NUMBER= 0.3775			
SHAFT POWER = 12992.2				SPECIFIC WORK = 171.68										
LAST STG M1 ABS =0.7840				LAST STG M1 REL = 0.3711			LAST STG M2 REL =0.8297				LAST STG M2 ABS = 0.4156			
HUB:	LAST STG M1 ABS =0.8937			LAST STG M1 REL = 0.4852			LAST STG M2 REL =0.7713				LAST STG M2 ABS = 0.3981			
	STATOR EXIT ANGLE = 67.95			ROTOR INLET ANGLE = 46.25			ROTOR EXIT ANGLE =-63.50				STAGE EXIT ANGLE = -30.16			
TIP:	LAST STG M1 ABS = 0.7042			LAST STG M1 REL = 0.3289			LAST STG M2 REL = 0.8626				LAST STG M2 ABS = 0.3764			
	STATOR EXIT ANGLE = 62.18			ROTOR INLET ANGLE = 2.30			ROTOR EXIT ANGLE =-66.51				STAGE EXIT ANGLE = -24.04			

STAGE 1- 2 MEANLINE SLOPE = 14.99 DEG BASED ON MID ASPECT-RATIO BLADING

the specified input requirements, while the other two indicate the nonexistence of a hub or tip solution satisfying free-vortex conditions. These messages are presented in this section, and their causes are discussed.

(1) **INSUFFICIENT ENERGY**—This message is caused by the computed turbine exit total temperature being less than zero.

It indicates that the turbine specific work requirement is greater than the energy available in the gas. Therefore, either the specified shaft power must be decreased or the specified flow must be increased.

(2) INSUFFICIENT IDEAL ENERGY—This message is caused by the computed ideal energy being more than that available



from an infinite expansion of the gas. It indicates that the computed efficiency is too low to yield a valid solution. Corrective action includes decreasing power, increasing flow, using more stages, or perhaps using a different velocity diagram.

(3) **NEGATIVE TEX**—This message is caused by the computed turbine exit static temperature being less than zero. It indicates a low value of turbine exit total temperature and/or a high value of turbine exit velocity. Corrective action could be the same as for message (2) or decreasing a too high value of stage exit to average meridional kinetic energy ratio (E).

(4) **INSUFFICIENT EXIT AREA**—This message is caused by the computed exit area being larger than that available in the turbine. Such a situation can be remedied in many ways, including increasing exit diameter, decreasing stator exit angle, increasing inlet pressure, decreasing mass flow, and increasing stage exit to average meridional kinetic energy ratio.

(5) **INSUFFICIENT INLET AREA**—This message is caused by the computed inlet area being larger than that available in the turbine. Corrective measures are the same as for message (4).

(6) **NEGATIVE T AT STATOR HUB EXIT**—This message is caused by the stator-exit hub static temperature being less than zero as a result of a very high absolute velocity. Although a non-free-vortex design (not available from this program) might not result in a similar condition, it is extreme enough to indicate that the mean-section design may require change such as a larger diameter, smaller stator angle, or more stages. This condition does not interrupt the mean section computations.

(7) **NEGATIVE T AT ROTOR TIP EXIT**—This message is caused by the rotor-exit tip static temperature being less than zero as a result of a very high absolute velocity. Corrective measures are the same as for message (6) above.

## References

1. Glassman, A.J.: Computer Program for Preliminary Design Analysis of Axial-Flow Turbines. NASA TN D-6702, 1972.
2. Stewart, W.L.: A Study of Axial-Flow Turbine Efficiency Characteristics in Terms of Velocity Diagram Parameters. ASME Paper 61-WA-37, 1961.
3. Glassman, A.J., ed.: Turbine Design and Application. Vol. One. NASA SP-290, 1972.
4. Timko, L.P.: Energy Efficient Engine High Pressure Turbine Component Test Performance Report. (R82AEB406, General Electric Co.; NASA Contract NAS3-20643), NASA CR-168289, 1984.
5. Whitney, W.J.: Cold-Air Investigation of a 3 1/2-Stage Fan-Drive Turbine with a Stage Loading Factor of 4 Designed for an Integral Lift Engine. 2: Performance of 2-, 3- and 3 1/2-Stage Configurations. NASA TM X-3482, 1977.
6. Bider, B., et al.: Cold-Air Investigation of a Turbine for High-Temperature-Engine Application. NASA TN D-4389.
7. Whitney, W.J.; Schum, H.J.; and Behning, F.P.: Cold-Air Investigation of a Turbine for High-Temperature-Engine Application. 4: Two-Stage Turbine Performance. NASA TN D-6960, 1972.
8. Kofskey, M.G.; Nusbaum, W.J.; and Haas, J.E.: Turbine for Ordinance Turbojet Engine. 2. Cold-Air Performance With Opened Stator. NASA TN D-7626.
9. Moffitt, T.P., et al.: Design and Cold-Air Test of Single-Stage Uncooled Turbine With High Work Output. NASA TP-1680, 1980.
10. Whitney, W.J., et al.: Cold-Air Investigation of 4 1/2-Stage Turbine With Stage Loading Factor of 4.66 and High Specific Work Output. 1: Overall Performance. NASA TM X-3498, 1977.
11. Stabe, R.G.; Whitney, W.J.; and Moffitt, T.P.: Performance of a High-Work Low Aspect Ratio Turbine Tested With a Realistic Inlet Radial Temperature Profile. NASA TM-83655, 1984.
12. Gardner, W.B.: Energy Efficient Program: High Pressure Turbine Uncooled Rig Technology Report. (PWA-5594-92, Pratt & Whitney Aircraft; NASA Contract NAS3-20646), NASA CR-165149, 1979.
13. Bridgeman, M.J.; Cherry, D.G.; and Pedersen, J.: NASA/GE Energy Efficient Engine Low Pressure Turbine Scaled Test Vehicle Performance Report. (R83AEB143, General Electric Co.; NASA Contract NAS3-20643), NASA CR-168290, 1983.

National Aeronautics and Space Administration  
Lewis Research Center, Cleveland, Ohio 44135  
August 12, 1991

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE February 1992		3. REPORT TYPE AND DATES COVERED Contractor Report
4. TITLE AND SUBTITLE  Computer Code for Preliminary Sizing Analysis of Axial-Flow Turbines			5. FUNDING NUMBERS  WU-505-69-50 G NAG3-1165	
6. AUTHOR(S)  Arthur J. Glassman				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  University of Toledo Department of Chemical Engineering Toledo, Ohio 43606			8. PERFORMING ORGANIZATION REPORT NUMBER  E-6150	
9. SPONSORING/MONITORING AGENCY NAMES(S) AND ADDRESS(ES)  National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135-3191			10. SPONSORING/MONITORING AGENCY REPORT NUMBER  NASA CR-4430	
11. SUPPLEMENTARY NOTES Project Manager, Daniel C. Mikkelson, Aeropropulsion Analysis Office, NASA Lewis Research Center. Arthur J. Glassman, The University of Toledo, Toledo, Ohio 43606 and Resident Research Associate at NASA Lewis Research Center. Final Report				
12a. DISTRIBUTION/AVAILABILITY STATEMENT  Unclassified - Unlimited Subject Category 07			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  This mean-diameter flow analysis uses a stage-average velocity diagram as the basis for the computation of efficiency. Input design requirements include power or pressure ratio, flow rate, temperature, pressure, and rotative speed. Turbine designs are generated for any specified number of stages and for any of three types of velocity diagrams (symmetrical, zero exit swirl, or impulse) or for any specified stage swirl split. Exit turning vanes can be included in the design. The program output includes inlet and exit annulus dimensions, exit temperature and pressure, total and static efficiencies, flow angles, and last-stage absolute and relative Mach numbers. This report presents the analysis method and a description of the computer program input and output with sample cases. The analysis and code presented herein are modifications of those described in NASA TN-D-6702. These modifications improve modeling rigor and extend code applicability.				
14. SUBJECT TERMS  Turbines; Axial flow			15. NUMBER OF PAGES 16	
			16. PRICE CODE A03	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT	